# FUNCTIONS AND GRAPHS

Precalculus

Chapter 1

- This Slideshow was developed to accompany the textbook
  - *Precalculus*
  - *By Richard Wright*
  - *https://www.andrews.edu/~rwright/Precalculus-RLW/Text/TOC.html*
- Some examples and diagrams are taken from the textbook.

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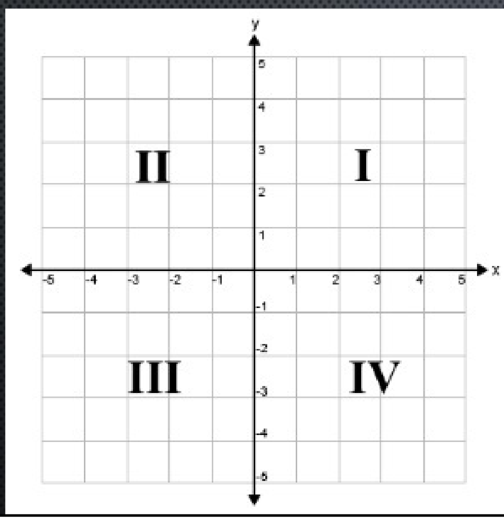
# 1-01 THE CARTESIAN PLANE

In this section, you will:

- Plot points in the cartesian coordinate system.
- Find the distance between two points.
- Find the midpoint between two points.

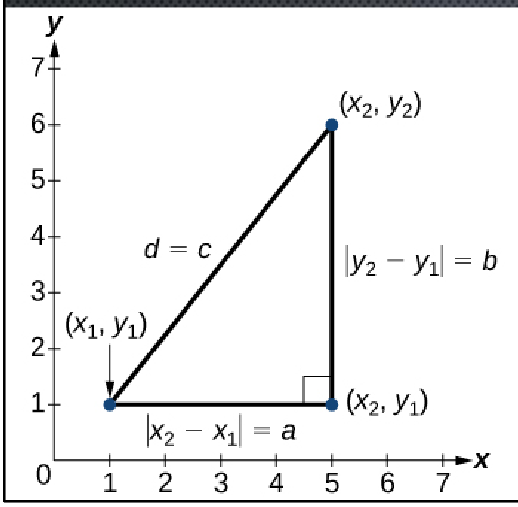
## 1-01 THE CARTESIAN PLANE




- Cartesian Plane
  - Four quadrants
- Point is (x, y)

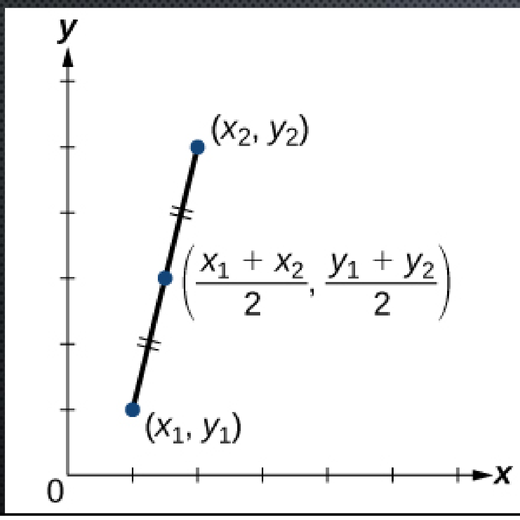
- Graph A(3, 2)
- Graph B(-1, 4)

# 1-01 THE CARTESIAN PLANE




- Distance formula
  - Pythagorean Theorem
  - $a^2 + b^2 = c^2$
  - $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$
- $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

## 1-01 THE CARTESIAN PLANE




- Midpoint formula
  - Average of the points (mean)
- $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

# 1-01 THE CARTESIAN PLANE

- Find the (a) distance and (b) midpoint between (-1, 3) and (2, -5)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{\left(2 - (-1)\right)^2 + (-5 - 3)^2}$$
$$d = \sqrt{3^2 + (-8)^2}$$
$$d = \sqrt{9 + 64}$$
$$d = \sqrt{73}$$

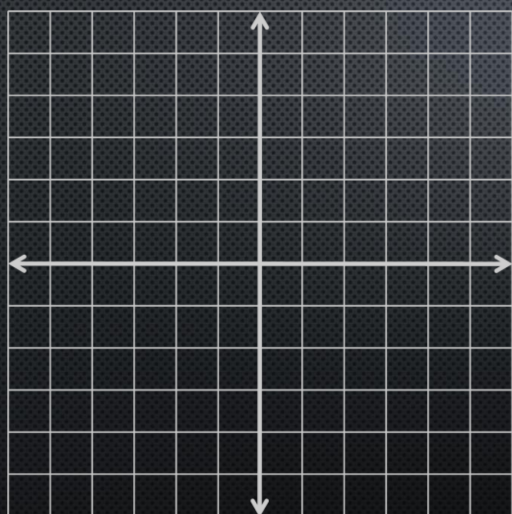
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$M = \left(\frac{-1 + 2}{2}, \frac{3 + (-5)}{2}\right)$$
$$M = \left(\frac{1}{2}, -\frac{2}{2}\right)$$
$$M = \left(\frac{1}{2}, -1\right)$$

# 1-02 GRAPHS

In this section, you will:

- Graph equations by plotting points.
- Graph equations with a graphing utility.
- Find the $x$- and $y$-intercepts.
- Graph circles.

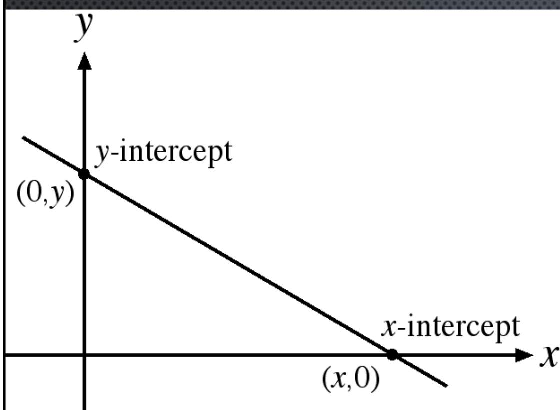
## 1-02 GRAPHS



- Basic graphing method
  - Make a table
  - Choose $x$, Calculate $y$
- Graph $y = 3 - 0.5x$

(x, y)
(-2, 4)
(-1, 3.5)
(0, 3)
(1, 2.5)
(2, 2)

## 1-02 GRAPHS




- Intercepts
- Point where a graph crosses the axes

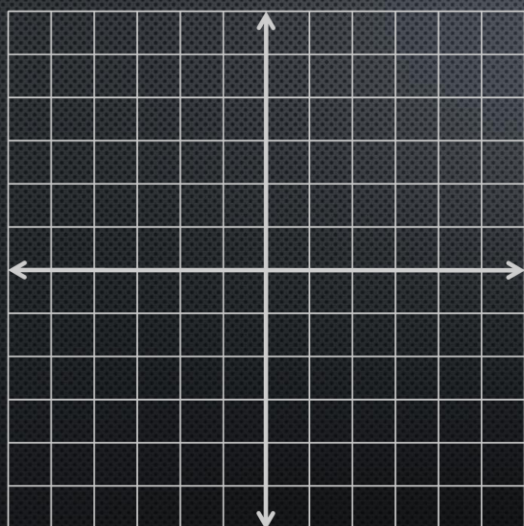
- To find the intercepts
- $x$-intercept
  - Let $y = 0$ and solve for $x$
- $y$-intercept
  - Let $x = 0$ and solve for $y$

## 1-02 GRAPHS

- Find the intercepts of $y = 2x^2 + 2$

No real x-int
y-int: (0, 2)

## 1-02 GRAPHS



- Circles
- $(x-h)^2+(y-k)^2=r^2$
  - where $(h,k)$ is the center
  - and $r$ is the radius
- Graph $(x+2)^2+(y-1)^2=4$

# 1-03 LINEAR EQUATIONS IN TWO VARIABLES

In this section, you will:

- Calculate and interpret slope.
- Write linear equations.
- Graph linear functions.

## 1-03 LINEAR EQUATIONS IN TWO VARIABLES

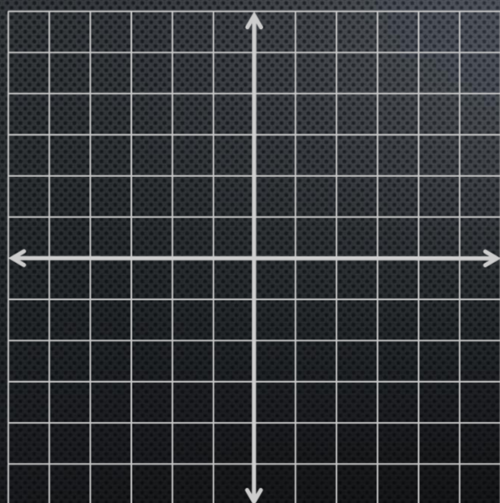
- Slope-intercept form
- $y = mx + b$
  - $m$ = slope (rate of change)
  - $(0, b)$ = $y$-intercept

- $y = b$ → horizontal line
- $x = a$ → vertical line

- To graph a line (shortcut)

1. Plot $y$-intercept
2. Follow the slope to get a couple more points
3. Draw a line through the points

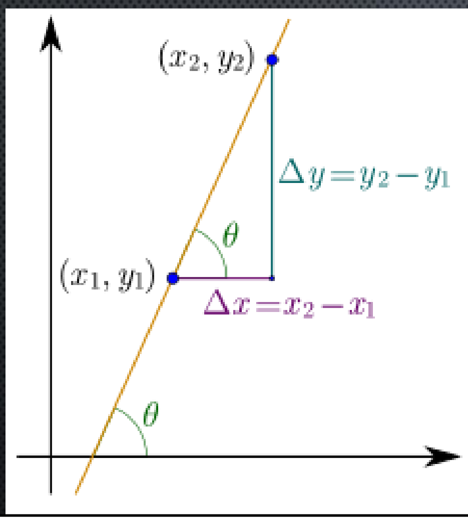
# 1-03 LINEAR EQUATIONS IN TWO VARIABLES



- Find the slope and $y$-int and graph $y = 3x - 4$

m=3
b=-4

# 1-03 LINEAR EQUATIONS IN TWO VARIABLES




- Slope
- $slope = \frac{rise}{run}$
- $m = \frac{y_2 - y_1}{x_2 - x_1}$
- If slope is
  - $m > 0$ → rises
  - $m = 0$ → horizontal
  - $m < 0$ → falls
  - $m$ undefined → vertical

## 1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Find the slope of the line passing through (-3, -2) and (1, 6)

$$m = \frac{6 - (-2)}{1 - (-3)}$$
$$m = \frac{8}{4} = 2$$

## 1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Write Linear Equations

1. Find slope ($m$)
2. Find a point on the line $(x_1, y_1)$
3. Use point-slope form $y - y_1 = m(x - x_1)$

- Find slope-intercept form of the line passing through (0, -2) with $m$ = 3.

y=3x-2

## 1-03 LINEAR EQUATIONS IN TWO VARIABLES

- Parallel and Penpendicular
- Parallel → same slope
- Perpendicular → slopes are negative reciprocals
  - $m_1 \cdot m_2 = -1$

- Find the equation of the line passing through (2, 1) and perpendicular to $4x - 2y = 3$.

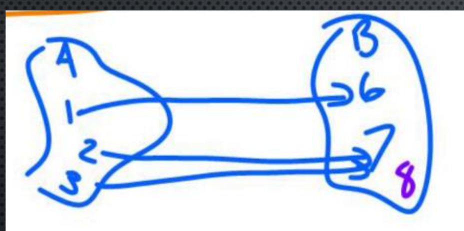
$$y = -\frac{1}{2}x + 2$$

# 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

In this section, you will:

- Determine whether a relation represents a function.
- Find input and output values of a function.
- Find the domain of a function.
- Evaluate piecewise functions.

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION




- Relation
  - Rule that relates 2 quantities
- Function
  - Special relation
  - A function *f* from set A to set B is a relation that assigns each element *x* in set A to exactly one element in set B
  - Set A: input domain
  - Set B: output range

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Is this a function?

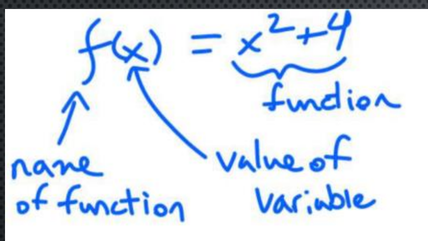
| | | | | | |
|---|---|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 |
| y | -8 | -1 | 0 | 1 | 8 |

- $x^2 + y = 4$
- $x + y^2 = 16$

Yes, each x is matched to exactly one y
Yes, each x gives exactly one y-value $y = -x^2 + 4$
No, some x give two y-values $y = \pm\sqrt{-x + 16}$

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Functional Notation




- Evaluate
- $f(y) = 3 - \sqrt{y}$
- $f(4)$
- $f(4x^2)$

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Piecewise functions
  - Function made of more than one function with specific domains
  - $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

- Evaluate
- $f(-1)$
- $f(2)$

f(-1)=-1
f(2)=6

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Domain of a function
  - Implied domain - all real numbers for which the expression is defined
- Interval notation
  - [ ] means =
  - ( ) means ≠
  - (2, 7] means $2 < x \leq 7$

- What is the domain?
- $h(t) = \frac{4}{t}$
- $f(x) = \sqrt{5x - 8}$

t≠0, (-∞,0) ∪ (0,∞)
x≥8/5, [8/5, ∞)

## 1-04 FUNCTIONS AND FUNCTIONAL NOTATION

- Difference Quotient
- $\frac{f(x+h)-f(x)}{h}$
- Simplify the difference quotient for $f(x) = 2x + 1$

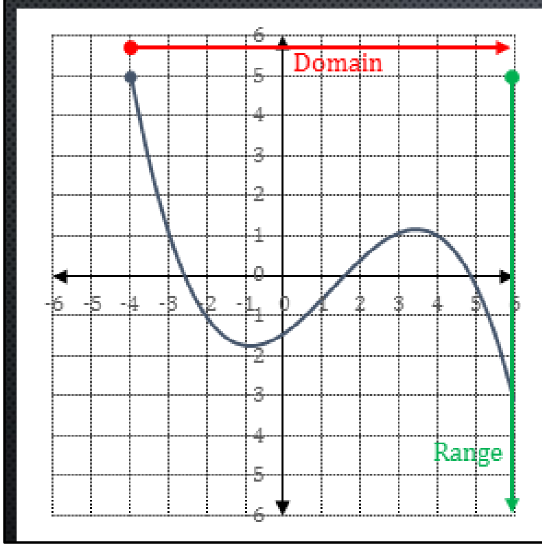
2

# 1-05 GRAPHS OF FUNCTIONS

In this section, you will:

- Find domain and range from graphs.
- Determine whether graphs represent functions.
- Find zeros of functions.
- Find the average rate of change of a function.
- Analyze graphs to determine when the graph is increasing, decreasing, or constant.

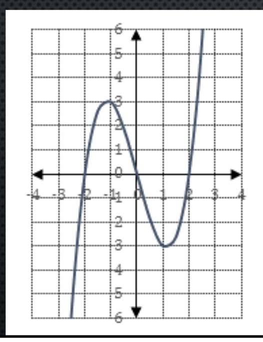
# 1-05 GRAPHS OF FUNCTIONS

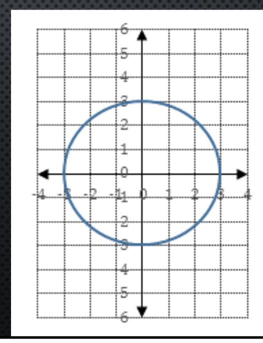



- Find the domain and range from a graph
- Domain: part of x-axis covered by graph
- Range: part of y-axis covered by graph

## 1-05 GRAPHS OF FUNCTIONS

- Vertical Line Test
- A graph represents a function if no vertical line can touch 2 points on the graph





Function
Not function

## 1-05 GRAPHS OF FUNCTIONS

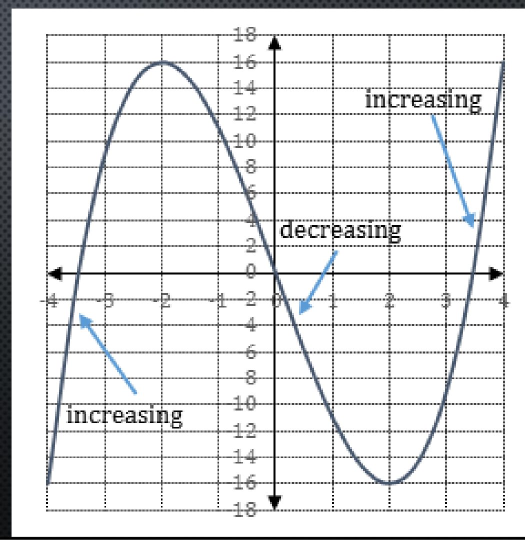
- Zeros of a function
  - $x$-value such that $f(x) = 0$
  - $x$-intercepts
- To find, make $f(x) = 0$ and solve for $x$

- Find the zeros of $f(x) = 2x^2 - 7x - 30$

-5/2, 6

# 1-05 GRAPHS OF FUNCTIONS

- Increasing (rises from left to right)
- Decreasing (falls from left to right)
- Constant (horizontal)
- Relative minimum (lowest point in area)
- Relative maximum (highest point in area)




Increasing (-∞,-2) ∪ (2, ∞)
Decreasing (-2, 2)
Minimum (2, -16)
Maximum (-2 16)

## 1-05 GRAPHS OF FUNCTIONS

- Rate of Change
- Average rate of change = slope between 2 points

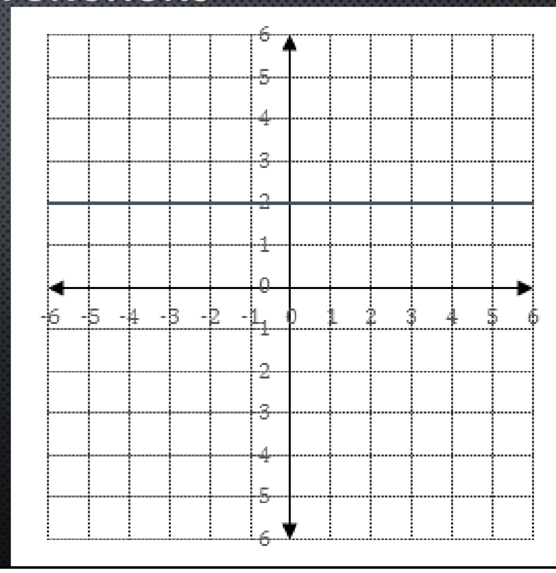
# 1-06 GRAPHS OF PARENT FUNCTIONS

In this section, you will:

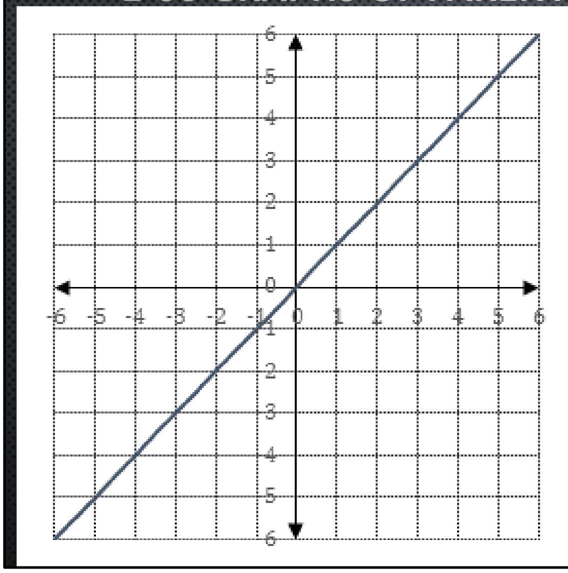
- Identify the graphs of parent functions.
- Graph piecewise functions.

## 1-06 GRAPHS OF PARENT FUNCTIONS

- **constant** function $f(x) = c$,
- Domain is all real numbers.
- Range is the set $\{c\}$ that contains this single element.
- Neither increasing or decreasing.
- Symmetric over the $y$-axis



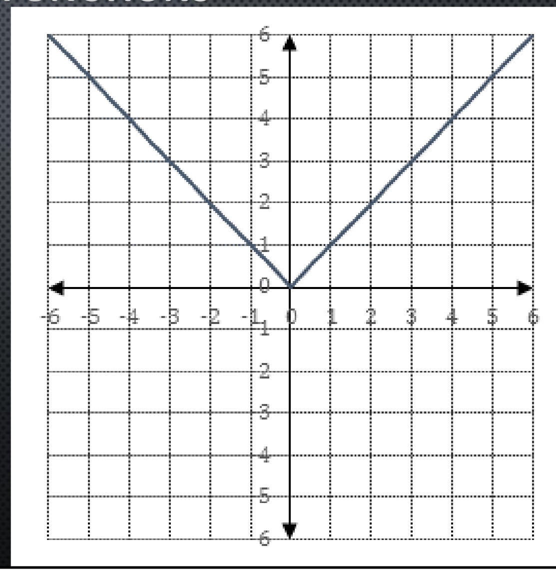
# 1-06 GRAPHS OF PARENT FUNCTIONS



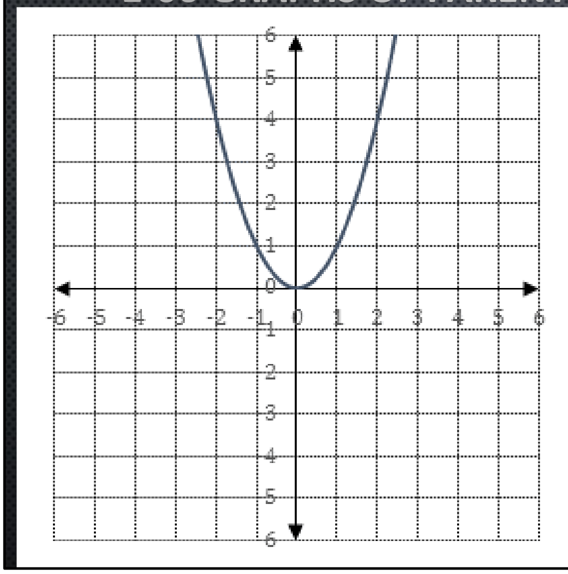

- **identity** function $f(x) = x$,
- Domain is all real numbers.
- Range is all real numbers.
- Increases from $(-\infty, \infty)$.
- Symmetric about the origin.

## 1-06 GRAPHS OF PARENT FUNCTIONS

- **absolute value** function $f(x) = |x|$,
- Domain is all real numbers.
- Range is $[0, \infty)$.
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric over the $y$-axis



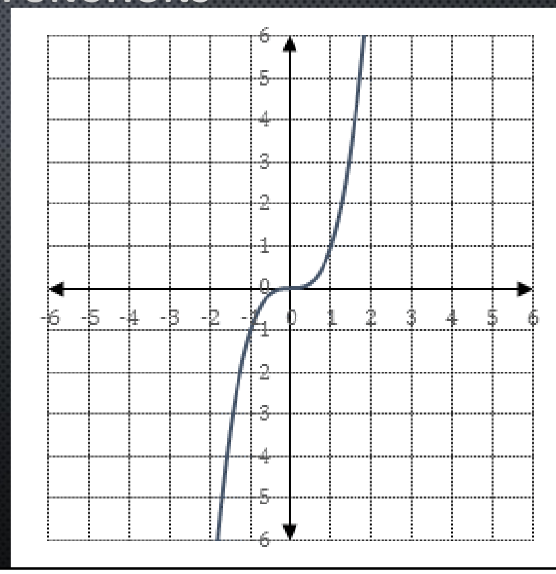
# 1-06 GRAPHS OF PARENT FUNCTIONS



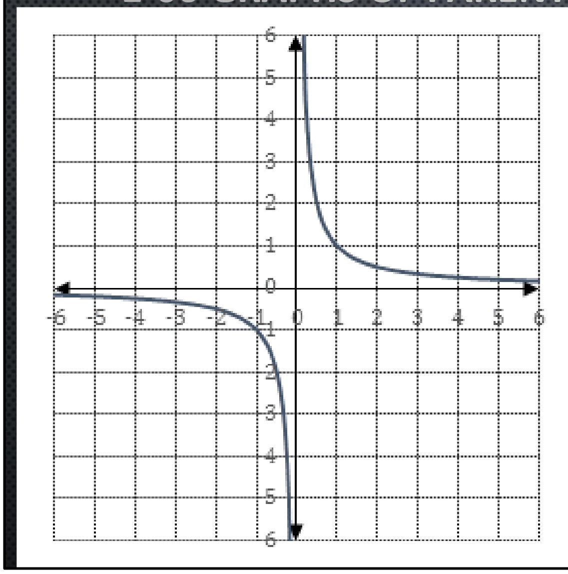

- **quadratic** function $f(x) = x^2$,
- Domain is all real numbers.
- Range is only nonnegative real numbers, $[0, \infty)$.
- Decreasing over $(-\infty, 0)$ and increasing on $(0, \infty)$.
- Symmetric over the $y$-axis.

## 1-06 GRAPHS OF PARENT FUNCTIONS

- **cubic** function $f(x) = x^3$,
- Domain is all real numbers.
- Range is all real numbers.
- Increasing on $(-\infty, \infty)$.
- Symmetric about the origin.



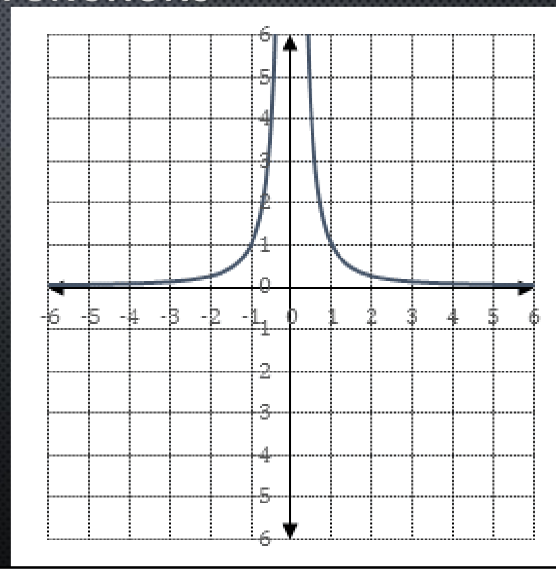
# 1-06 GRAPHS OF PARENT FUNCTIONS



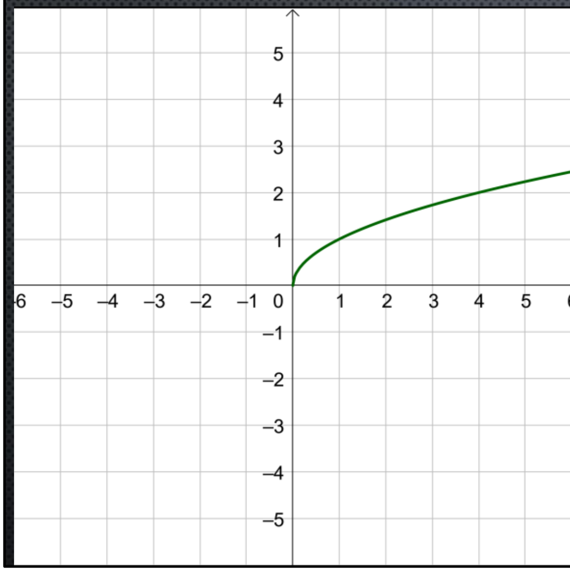

- **reciprocal** function $f(x) = \frac{1}{x}$,
- Domain is all real numbers except 0, $\{x | x \neq 0\}$.
- Range is all real numbers except 0, $\{y | y \neq 0\}$.
- Decreasing on $(-\infty, 0)$ and $(0, \infty)$.
- Symmetric about the origin and over the lines $y = x$ and $y = -x$.

## 1-06 GRAPHS OF PARENT FUNCTIONS

- **reciprocal squared** function $f(x) = \frac{1}{x^2}$,
- Domain is all real numbers except 0, $\{x|x \neq 0\}$.
- Range is only positive real numbers, $(0, \infty)$.
- Increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.
- Symmetric over the $y$-axis.



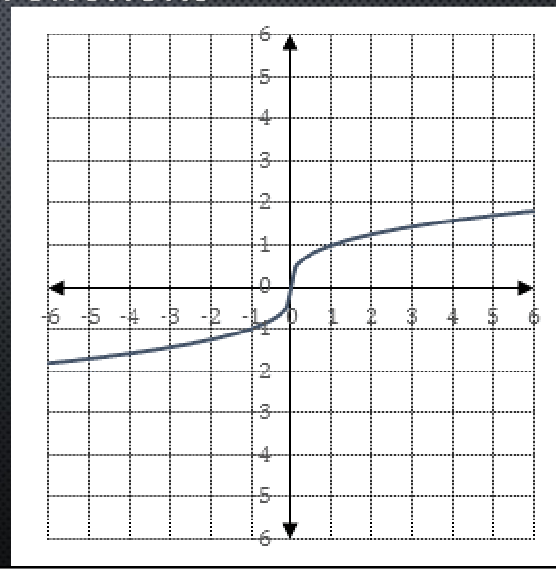
## 1-06 GRAPHS OF PARENT FUNCTIONS



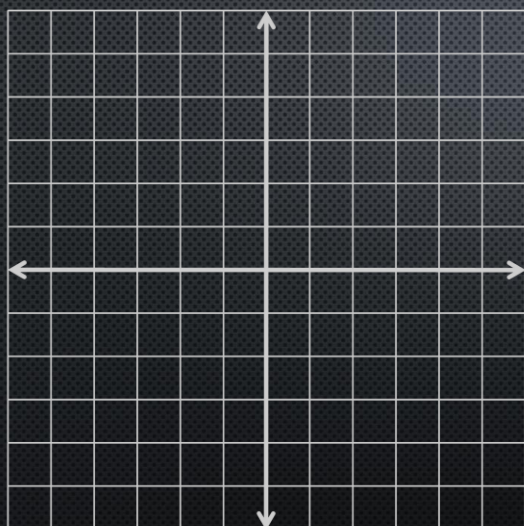

- **square root** function $f(x) = \sqrt{x}$,
- Domain is 0 or greater, $[0, \infty)$.
- Range is 0 or greater, $[0, \infty)$.
- Increasing on $(0, \infty)$.
- No symmetry.

# 1-06 GRAPHS OF PARENT FUNCTIONS

- **cube root** function $f(x) = \sqrt[3]{x}$,
- Domain is all real numbers.
- Range is all real numbers.
- Increasing over $(-\infty, \infty)$.
- Symmetric about the origin.



# 1-06 GRAPHS OF PARENT FUNCTIONS



- Piecewise Functions
- At the boundary,
  - If equal → solid dot
  - If not equal → open dot
- Graph $g(x) = \begin{cases} 3x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$

# 1-07 TRANSFORMATIONS OF FUNCTIONS

In this section, you will:

- Graph functions with translations.
- Graph functions with reflections.
- Graph functions with stretches and shrinks.
- Perform a sequences of transformations.

## 1-07 TRANSFORMATIONS OF FUNCTIONS

- Translations (shift)
  - Moves the graph
- Horizontal
  - $h(x) = f(x - c)$
  - $c$ shifts right
- Vertical
  - $h(x) = f(x) + d$
  - $d$ shifts up

- For $f(x) = |x|$, write a function with a vertical shift of 3 down and 2 right.

$$h(x) = |x - 2| - 3$$

## 1-07 TRANSFORMATIONS OF FUNCTIONS

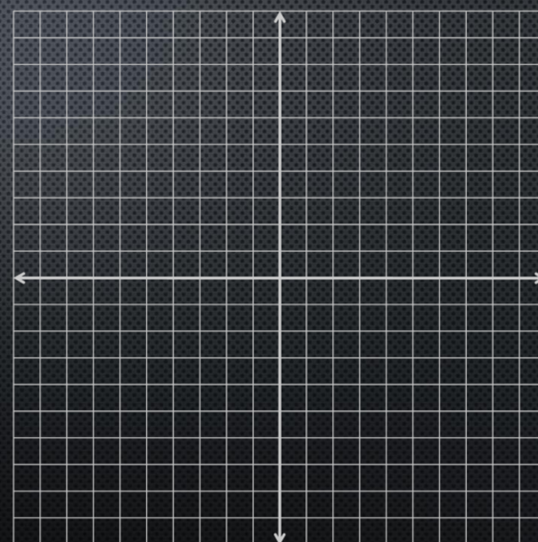
- Reflections
- $x$-axis
  - Vertical
  - $h(x) = -f(x)$
- $y$-axis
  - $h(x) = f(-x)$
  - Horizontal
- Dilations
  - Stretch/Shrink
- Horizontal
  - $h(x) = f(bx)$
  - Stretch by $\frac{1}{b}$
- Vertical
  - $h(x) = af(x)$
  - Stretch by $a$

## 1-07 TRANSFORMATIONS OF FUNCTIONS

- Put it all together
- $h(x) = af(bx - c) + d$
  - $a$ = vertical stretch
  - $\frac{1}{b}$ = horizontal stretch
  - $c$ = horizontal shift right
  - $d$ = vertical shift up

## 1-07 TRANSFORMATIONS OF FUNCTIONS

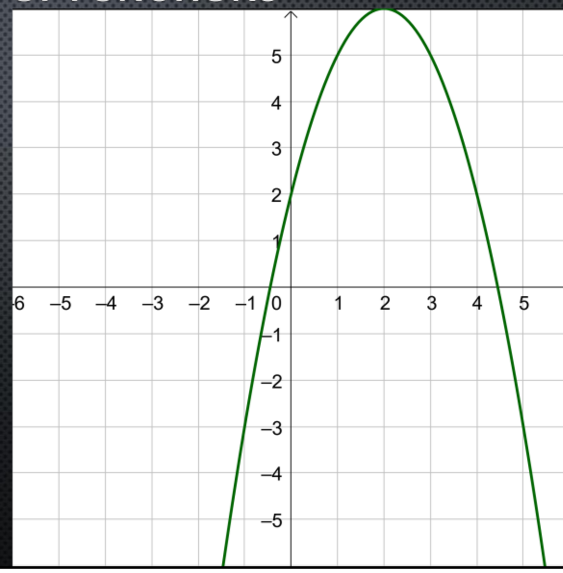
- Given $g(x) = 2 - (x + 5)^2$
- Identify the parent function
- Describe the transformations
- Sketch the graph
- Use functional notation to write $g$ in terms of $f$



Quadratic
Reflection over x-axis, shift 5 left, shift 2 up
Graph
$g(x) = -f(x + 5) + 2$

## 1-07 TRANSFORMATIONS OF FUNCTIONS

- Write the function for




Parent function is $f(x)=x^2$
Shift right 2: $c=2$
Shift up 6: $d=6$
Reflect over x-axis: $-f(x)$
$g(x)=-(x-2)^2+6$

# 1-08 COMBINATIONS OF FUNCTIONS

In this section, you will:

- Combine functions using algebraic operations.
- Create a composition of functions.

## 1-08 COMBINATIONS OF FUNCTIONS

- Add
  - $(f+g)(x) = f(x) + g(x)$
- Subtract
  - $(f-g)(x) = f(x) - g(x)$
- Multiply
  - $(fg)(x) = f(x)g(x)$
- Divide
  - $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

- If $f(x) = x + 2$ and $g(x) = x - 2$, find
- $(f+g)(x)$
- $(f-g)(x)$
- $(fg)(x)$
- $\left(\frac{f}{g}\right)(x)$

2x
4
$x^2$-4
(x+2)/(x-2)

## 1-08 COMBINATIONS OF FUNCTIONS

- Composition
- $(f \circ g)(x) = f(g(x))$
- Substitute $g$ into $f$

- If $f(x) = x^2$ and $g(x) = x - 1$, find
- $f \circ g$

- $g \circ f$

- Domain of $(f \circ g)$ is all $x$ in domain of $g$ such that $g(x)$ is in the domain of $f$.
- $x \rightarrow g \rightarrow f$

- If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$, find the domain of $f \circ g$

$x^2$-2x+1
$x^2$-1

Domain of g: $x \neq 0$
Domain of f: $x \geq 0$
Let g=domain of f (or excluded domain) and solve for x
$1/x \geq 0$ → true when $x>0$
Answer: $x>0$

## 1-08 COMBINATIONS OF FUNCTIONS

- Decompose
- Find $f(x)$ and $g(x)$ so that $(f \circ g)(x) = h(x)$
- Pick a portion to be g(x), then replace that with x to get f(x)
- Decompose $h(x) = 2|x + 3|$

- Decompose $h(x) = \sqrt[3]{\frac{8-x}{5}}$

g(x)=x+3; f(x)=2|x|

g(x)=(8-x)/5; f(x)=$\sqrt[3]{(x)}$

# 1-09 INVERSE FUNCTIONS

In this section, you will:

- Verify that two functions are inverse functions.
- Find the domain and range on inverse functions.
- Find the inverse of a function.

## 1-09 INVERSE FUNCTIONS

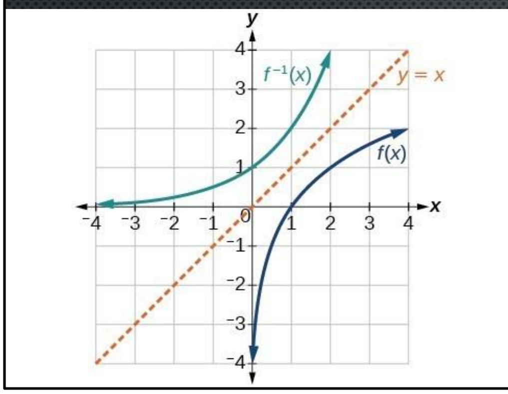
- Inverse functions
- Switch $x$ and $y$
- Switch inputs and outputs

- Verify inverses by showing
- $f(g(x)) = x$ and $g(f(x)) = x$

- Verify that $f(x) = 7x - 4$ and $g(x) = \frac{x+4}{7}$ are inverses

## 1-09 INVERSE FUNCTIONS

- Graphs of inverses
- Reflected over line $y = x$




- One-to-one
- A function is one-to-one if each $y$ corresponds to exactly one $x$.
- Passes the horizontal line test
- Inverse of a 1-to-1 is a function

## 1-09 INVERSE FUNCTIONS

- Finding inverses

1. Replace $f(x)$ with $y$
2. Switch $x$ and $y$
3. Solve for $y$
4. If you did step 1, replace $y$ with $f^{-1}(x)$

- Find the inverse of

$$f(x) = \sqrt[3]{10 + x}$$

$f^{-1}(x)=x^3-10$

## 1-09 INVERSE FUNCTIONS

- Find the inverse of $f(x) = x^2 - 2, x < 0$

$f^{-1}(x)=-\sqrt{(x+2)}$

# 1-10 MATHEMATICAL MODELING

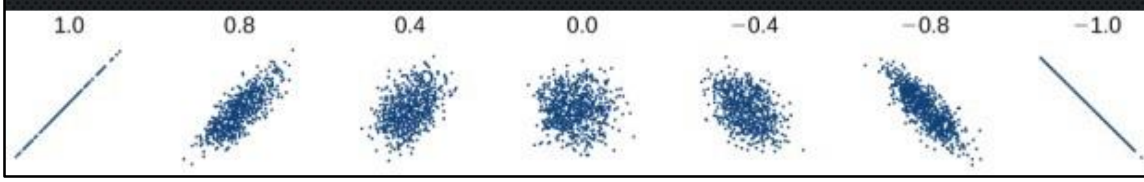
In this section, you will:

- Draw and interpret scatter plots.
- Find the best-fitting line using a graphing utility.
- Calculate variations.

## 1-10 MATHEMATICAL MODELING

- Mathematical modeling
- Find a function to fit data points

- Least squares regression (linear)
- Gives the best fitting line
- The amount of error is given by the correlation coefficient (*r*)



## 1-10 MATHEMATICAL MODELING

- Number (in 1000s) of female USAF personnel, $P$, on active duty

| Year | 2000 | 2001 | 2002 | 2003 | 2004 |
|---|---|---|---|---|---|
| $P$ | 66.8 | 67.6 | 71.5 | 73.5 | 73.8 |

- Find a model with $t$=0 being 2000

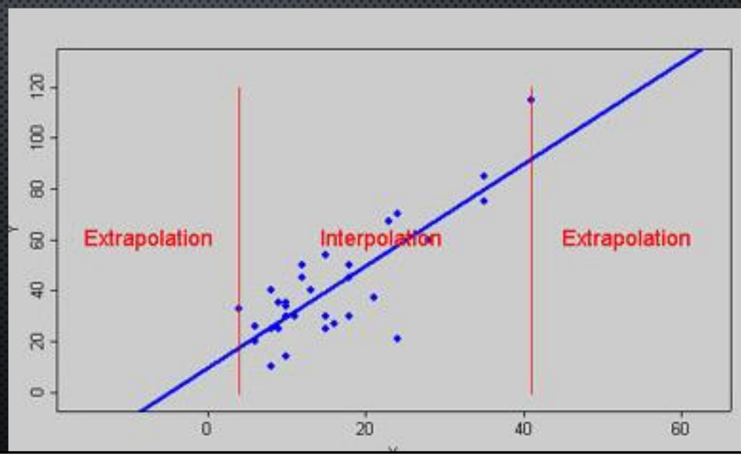
- On TI-graphing
- STAT ∨ Edit... and enter data
- STAT → CALC ∨ LinReg(ax+b)

P=1.99t+66.7

# 1-10 MATHEMATICAL MODELING

- Real-Life Problems
- Slope = rate of change

- Interpolation
  - Within data
  - Small error
- Extrapolation
  - Outside of data
  - Possibly huge error



## 1-10 MATHEMATICAL MODELING

- Variations
- Direct $y = ax$
  - $x \uparrow, y \uparrow$
- Inverse $y = \frac{a}{x}$
  - $x \uparrow, y \downarrow$
- Joint $z = axy$

- $a$ = constant of variation

- A company found the demand for its product varies inversely as the price of the product. When the price is $2.75, the demand is 600 units. Write an equation.

Inverse variation
d=1650/p